Chapter 14: *Game Theory*

Outline and Conceptual Inquiries

**The Game**
- Establishing Equilibrium

**Understanding the Strategic Form**
- **Prisoners’ Dilemma**
  - Why does a district attorney separate prisoners accused of a crime?
  - Why does joining a street gang have advantages?
  - Why are there multicellular creatures?
  - How should you raise your children (or rats)?

**Application: Game Theory is not for Playing Games**

**How to Play Sequential Games**
- Who should make the first move on a date?

**Preemption Games**
- When would you advocate tariffs and quotas?
- Why should a nondiscount store not lower prices when a discount store enters the market?
- When punishing a child, why does a parent say, “This will hurt me more than it will hurt you?”

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Appendix to Chapter 14
Using Mixed Strategies

How should you play the Rock, Paper, Scissors game?
Can randomly selecting your course of actions improve your happiness?

Application: Timing of Insider Trading in a Betting Market

Rational Behavior

Why do you drive through a green light?

Incomplete Information Games

Quid Pro Quo

Why did the United States and Vietnam argue over the shape of the table for peace negotiations?

Application: Quid Pro Quo? China’s Investment-for-Resource Swaps in Africa

Summary

1. A game is a model representing the strategic interdependence of agents (players) in a particular situation. Two methods for representing a game are strategic- and extensive-form games. The extensive form models everything in the strategic form plus additional information that may aid in eliminating certain outcomes from consideration.

2. The classic example of a simultaneous game is Prisoners’ Dilemma, which illustrates how cooperation can improve the welfare of all agents. Generally, to facilitate cooperation, some type of enforcement is required that rewards good behavior and punishes bad behavior.

3. In many economic situations, agents interact by moving sequentially rather than simultaneously. A game tree may be used to describe the extensive form of these sequential games. Backward induction is used for determining the solutions.

4. The agent who acts first generally has the advantage. Games that model such behavior are called preemption games and may be used for describing firm entry into a market, market niches, and threats.

5. (Appendix) In many economic situations, the optimal solution is to prevent other players from predicting your strategy. Thus, mixed strategies can often improve a player’s payoff.

6. (Appendix) Games with incomplete agent information as a result of some information asymmetries are called Bayesian games. Similar to mixed-strategy games, probabilities associated with payoffs are used for determining solutions.

7. (Appendix) Single issues represented by one game are not often resolved in isolation from other decisions. Generally, a multitude of issues and decisions require determining a joint outcome. A method for resolving these issues is quid pro quo—something for something else.

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Key Concepts

backward induction
Bayesian game
Bayesian Nash equilibrium
collusion
complete information game
cooperative game theory
decision rule
dominant strategy
dynamic game
empty threats
exploiters
extensive form
finitely repeated game
game theory
game
game matrix
game tree
idle threats
imperfect competition
imperfect information game
infinitely repeated game
iterated deletion
market niche
maximin strategy
mixed strategies

Nash equilibrium
noncooperative game theory
payoff
payoff matrix
perfect information game
player
preemption games
principle of sequential rationality
Prisoners’ Dilemma
pure Nash equilibrium
pure strategies
quid pro quo
rationalizable strategy
root
security value
sequential games
strategic form
strategic interdependence
strategy
strategy pairs
tit-for-tat
trembling-hand game
trembling-hand Nash equilibrium
trigger
win-stay/lose-shift
TEST YOURSELF

Multiple Choice

1. Which of the following is not true in game theory?
   a. The optimal actions of a player may depend on what he expects other players to do
   b. The number of players in a game is limited to two
   c. Players determine their strategies by examining the payoffs associated with the various outcomes of the game
   d. Games may have multiple equilibria or no equilibrium.

2. A game matrix is generally used if the game
   a. Is shown in strategic form
   b. Involves more than two players
   c. Is played sequentially
   d. Is shown in extensive form.

3. A dominant strategy occurs if
   a. Both players choose the same strategy
   b. One player has a strategy that is preferred to other strategies no matter what the other players’ strategies are
   c. The payoff to a strategy depends on the choice made by the player’s opponent.
   d. All players have a strategy that is preferred to other strategies no matter what the other players’ strategies are.

4. When each player’s selected strategy is his preferred response to the strategies actually played by all other players, there is a
   a. Tit-for-tat equilibrium
   b. Nash equilibrium
   c. Dominant strategy
   d. Cooperative outcome.

5. Two players each have a red card and a black card in their hands. They must choose one and raise it in the air. The payoff matrix for this game is

   Player B
   
<table>
<thead>
<tr>
<th></th>
<th>Red</th>
<th>Black</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td>(5, 30)</td>
<td>(10, 12)</td>
</tr>
<tr>
<td>Black</td>
<td>(-2, 10)</td>
<td>(8, 15)</td>
</tr>
</tbody>
</table>

   Which of the following is correct?
   a. Neither player has a dominant strategy
   b. Player A has one dominant strategy, while player B has none
   c. Both players have one dominant strategy
   d. Player B has one dominant strategy, while player A has none.
6. Refer to Question 5. The expected outcome of the game is that
   a. Both players will choose black
   b. Player A will choose black, while Player B chooses red
   c. Both players will choose red
   d. Player A will choose red, while Player B chooses black.

7. Tony and Marj must decide where to vacation. They face two possible choices: a cruise or a resort. Each can choose either destination and can choose to vacation alone. Assume that they make the choice simultaneously. The payoff matrix for this game is

<table>
<thead>
<tr>
<th></th>
<th>Cruise</th>
<th>Resort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marj</td>
<td>(3, 5)</td>
<td>(2, 2)</td>
</tr>
<tr>
<td>Cruise</td>
<td>(1, 1)</td>
<td>(5, 3)</td>
</tr>
</tbody>
</table>

   How many equilibria are present in this game?
   a. 0
   b. 1
   c. 2
   d. 3.

8. Refer to Question 7. Which of the following is correct?
   a. There will be a Nash equilibrium if the husband goes on a cruise and the wife visits the resort
   b. Traveling alone is preferable for the husband
   c. If the wife chooses first, the husband will choose the resort
   d. If the husband chooses first, the wife will choose the resort.

9. The Prisoners’ Dilemma refers to games where
   a. Neither player has a dominant strategy
   b. The game has multiple equilibria
   c. The dominant outcome leads to a lower payoff than the cooperative outcome
   d. Both players choose the same strategy.

10. The market for sushi in a small California town contains only two firms (Uncooked and Raw). The owners of the two firms decide to fix the price of sushi. The two firms must then decide whether to abide by their agreement or cheat. The payoff matrix is

<table>
<thead>
<tr>
<th></th>
<th>Cheat</th>
<th>Abide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncooked</td>
<td>(40, 40)</td>
<td>(80, 0)</td>
</tr>
<tr>
<td>Abide</td>
<td>(0, 80)</td>
<td>(45, 45)</td>
</tr>
</tbody>
</table>

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Based on this information, cheating is a dominant strategy for
a. Neither
b. Both
c. Raw, but not Uncooked
d. Uncooked, but not Raw.

11. Refer to Question 10. Suppose Raw decides to use a tit-for-tat strategy. Which of the following is least likely to occur?
   a. Raw will begin by abiding by the agreement
   b. The probability that Uncooked will cheat is now higher than if Raw followed its dominant strategy
   c. Raw will follow the previous move by Uncooked.
   d. If Uncooked always abides by the agreement, both firms will receive profits of 45 each.

12. A win-stay/lose-shift policy is one where a player
   a. Receives higher payoffs than he would using a tit-for-tat strategy
   b. Follows his opponent’s move from the previous period
   c. Changes his strategy only when he loses
   d. Always follows his dominant strategy.

13. A game in which agents who move first have an advantage is called a(n)
   a. Nash game
   b. Exploiter game
   c. mixed-strategy game
   d. Preemption game.

14. Suppose two big-box stores (A and B) are considering entering a market. The payoff matrix for this game is

<table>
<thead>
<tr>
<th></th>
<th>Entry</th>
<th>No Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entry</td>
<td>(−10, −8)</td>
<td>(100, 40)</td>
</tr>
<tr>
<td>No Entry</td>
<td>(20, 60)</td>
<td>(10, 10)</td>
</tr>
</tbody>
</table>

   Which of the following is correct?
   a. If both firms must decide simultaneously, they will both enter the market
   b. If Firm A gets to make its decision first, Firm B will not choose to enter the market
   c. If Firm B gets to make its decision first, Firm A will choose to enter the market
   d. They will cooperate and both not enter the market.

15. Refer to Question 14. Suppose both firms can bid for the right to make its decision first. If so, Firm A would be willing to pay up to _______ and Firm B would be willing to pay up to _______.
   a. $80; $20
   b. $100; $60
   c. $90; $50
   d. $20; $40.
Short Answer

1. Explain the difference between strategic- (normal-) form and extensive-form games. Which types of diagrams are most suitable for each of these forms?

2. Suppose Doug and Sandra are each flipping a quarter. If both tosses show the same side of the coin, Doug receives the quarters. If both tosses show different sides of the coin, Sandra receives the quarters. Construct a payoff matrix to represent this game.

3. What is a dominant strategy? If all players have dominant strategies and follow them, will the result be a Nash equilibrium? Explain.

4. Why is the Prisoners’ Dilemma referred to as a “dilemma”?

5. The market for sushi in a small California town contains only two firms: Uncooked and Raw. The owners of the two firms decide to fix the price of sushi. The two firms must then decide whether to abide by their agreement or cheat. The payoff matrix is

<table>
<thead>
<tr>
<th></th>
<th>Raw Abide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncooked Cheat</td>
<td>(40, 40)</td>
</tr>
<tr>
<td>Abide</td>
<td>(80, 0)</td>
</tr>
<tr>
<td>Abide  (0, 80)</td>
<td>(45, 45)</td>
</tr>
</tbody>
</table>

Can the owner of Uncooked improve the outcome of the game by promising the owner of Raw that he will abide by the agreement? Explain.

6. Tony and Marj must decide where to vacation. They face two possible choices, a cruise or a resort. Each can choose either destination and can choose to vacation alone. Assume that they make the choice simultaneously. The payoff matrix for this game is

<table>
<thead>
<tr>
<th></th>
<th>Cruise Resort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tony</td>
<td></td>
</tr>
<tr>
<td>Marj</td>
<td>Cruise (3, 5)</td>
</tr>
<tr>
<td></td>
<td>Resort (1, 1)</td>
</tr>
</tbody>
</table>

Are separate vacations Nash equilibria? Why or why not?

7. Suppose the Prisoners’ Dilemma game is played repeatedly. Explain how the outcomes would likely differ for the following two cases: (1) the game is to be played seven times; and (2) the game is to be played an infinite number of times.

8. A player (involved in a repeating Prisoners’ Dilemma game) is trying to decide between two strategies: tit-for-tat and win-stay/lose-shift. Explain what each of these strategies implies about the player’s moves. Can you advise this player on the best strategy to follow? Explain.
9. Suppose two neighbors (Hatfield and McCoy) are trying to decide how loud to play their home theaters at night. Hatfield gets off of work before McCoy and therefore turns his theater on first. The payoffs for this game can be represented by the following game tree:

```
Hatfield
  /  \                           (Hatfield, McCoy)
/    \        Loud              (5, 3)
|      /\      McCoy              \
|     /  \                        \
|    |    \                      (3, 5)
|    |     \                    (3, 5)
|    |      \                (1, 6) McCoy
|    |       |   Loud
|    |       +-----\       (4, 7)   Soft
|    |       |
|    |       |
|    |       |
|    |       |
|    |       |
|    |       |
|    |       |
|    |       |
|    |       |
|    |       |
|    |       |
|    |       |
|    |       |
```

Is there an equilibrium strategy? Construct a reduced game tree for this problem.

10. *(Appendix)* Explain how a player can use a maximin strategy.
Problems

1. Suppose two individuals (Erick and Gale) are working on a project together. They must each decide whether to work hard or to shirk. Suppose the payoff matrix for this game is

\[
\begin{array}{c|cc}
& \text{Work} & \text{Shirk} \\
\hline
\text{Erick} & (10, 10) & (2, 20) \\
\text{Shirk} & (20, 2) & (5, 5) \\
\end{array}
\]

a. Do either of the players have a dominant strategy? Explain.
b. Is there a cooperative outcome that would yield a higher payoff for the players? Explain. If so, how can these players increase the likelihood of reaching the cooperative agreement?

2. Two talk-radio stations (WYEA and WNAY) are competing for listeners in two time slots: 2:00–3:00am and 5:00–6:00pm. Each has two shows to fill up these time slots and can choose to place its more widely known show in the morning or the afternoon. The combination of decisions leads to the following payoff matrix in terms of ratings:

\[
\begin{array}{c|cc}
& \text{am} & \text{pm} \\
\hline
\text{WYEA} & (20, 30) & (18, 18) \\
\text{pm} & (15, 15) & (30, 10) \\
\end{array}
\]

a. Determine the Nash equilibria for this game, assuming the radio stations make their decisions simultaneously.
b. What will be the equilibrium if WYEA chooses its lineup first? What will be the equilibrium if WNAY chooses its lineup first?
c. Suppose the managers of the radio stations meet to coordinate their schedules. If the manager of WNAY promises to put its more well-known show in the afternoon, is this promise credible? Why or why not?

3. Suppose a deputy (Barney Fife) is pursuing a suspected burglar (Gomer). Gomer is at a stop sign and must determine which way to turn (right or left). If he turns right, he can cross the county line and escape. If he turns left, he can go to the garage of his apartment. Barney cannot see Gomer’s car, but understands the options he has. He can also turn right or left at the stop sign when he reaches it. If he turns right (and Gomer turned right), Barney will be able to catch up with Gomer. If he turns left (and Gomer turned left), he can also arrest Gomer. But, if Barney turns the wrong way, he will find himself in trouble. The payoff matrix is

\[
\begin{array}{c|cc}
& \text{Right} & \text{Left} \\
\hline
\text{Gomer} & (−2, 2) & (3, −4) \\
\text{Left} & (4, −1) & (−1, 3) \\
\end{array}
\]
a. Does either or both player have a dominant strategy? Explain.
b. Is there a Nash equilibrium? Explain.
c. *(Appendix)* Barney must estimate the probability that Gomer has turned right. What is the minimum probability that would induce Barney to turn right?

4. *(Appendix)* Two vehicles meet at the intersection of University and College. Each has two strategies: wait or go. The payoffs are shown in this matrix:

<table>
<thead>
<tr>
<th></th>
<th>Wait</th>
<th>Go</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle 1</td>
<td>(0, 0)</td>
<td>(1, 5)</td>
</tr>
<tr>
<td>Vehicle 2</td>
<td>(5, 1)</td>
<td>(−10, −10)</td>
</tr>
</tbody>
</table>

If both vehicles make their decisions simultaneously and follow a maximin strategy, what is the outcome? Is this a Nash equilibrium?

5. *(Appendix)* Consider a baseball game between the pitcher and batter. Assume the pitcher has two pitches: a fastball and a change-up. The change-up is a slower pitch that would be easy to hit if the batter knew it was coming. But, if the batter is expecting a fastball, the change-up pitch may fool him. The batter can hit the fastball, assuming that he swings early enough. Further assume there is only one pitch. The matrix is

<table>
<thead>
<tr>
<th></th>
<th>Fastball</th>
<th>Change-Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batter</td>
<td>(10, −12)</td>
<td>(−5, 3)</td>
</tr>
<tr>
<td>Fastball</td>
<td>(−6, 1)</td>
<td>(3, −4)</td>
</tr>
</tbody>
</table>

The batter and the pitcher want to be unpredictable, and therefore choose to employ a mixed strategy. What will be each player’s mixed strategy?