Chapter 19: *Intertemporal Choice and Capital Decisions*

Outline and Conceptual Inquiries

**Intertemporal Choice**
- Understanding Intertemporal Indifference Curves
  - *Are you patient or impatient?*
- Intertemporal Budget Constraint
  - Establishing an Equilibrium
    - *Are you a lender, a borrower, or a Polonius?*

**Developing Human Capital**
- Maximizing Utility with no Financial Markets: A Developing Country
- Maximizing Utility with Financial Markets (the Separation Theorem)
  - *Why do college students shop at electronic stores and boutiques?*

**Determination of the Interest Rate $i$**
- Real Interest Rate (Rate of Return) Versus Nominal Interest Rate

**Discounting the Future**
- Discounting in a Two-Period Model
  - *Is a dollar forthcoming a year from now worth as much as a dollar today?*
  - Application: Consumer Discount Rates for Energy-Using Durables

**Multi-Period Discounting**
- Maximizing Present Value
  - Application: Public Disaster Compensation Funds
- Infinite Horizon
- Net Present Value (NPV)
  - Application: Avoiding Real Estate Bubbles

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Appendix to Chapter 19
Comparative Statics: Interest Rate Change

Adjusted Slutsky Equation
Will a borrower always decrease consumption given a rise in interest rates?

Application: Taxation on Interest Income
Will a tax rate reduction on interest income increase savings?

Rate of Return: Telemarketing Fraud

Uncertain Investment Decisions: The Value of Waiting
Does the ability to wait have value?

Utility-Based Investments: Jewelry Investments
Is purchasing a diamond necessarily a rational investment?

Intertemporal Choice and Behavioral Economics

Summary

1. Determining the optimal level of capital inputs requires calculating the value of future returns associated with the capital input.

2. An impatient agent prefers current consumption to postponing consumption into the future. A patient agent is willing to postpone some current consumption for less of it in the future.

3. The Polonius point is where an agent is neither a borrower nor a lender but is consuming all of the income endowed within a time period during the time period.

4. The equilibrium levels of current and future consumption occur where the marginal rate of time preference is set equal to the time rate of substituting future consumption for current consumption.

5. With no financial markets, consumers’ optimal levels of consumption and investment in human capital are determined by the tangency between the indifference curve and the human-capital production function.

6. The ability to borrow and lend money separates the level of human-capital investment from the decision of optimal consumption across time periods. Such a separation is an application of the Separation Theorem.

7. The intersection of the supply and demand curves for future consumption will determine the equilibrium interest rate.

8. The observed market interest rate is called the nominal interest rate and is determined by both the real interest rate and the expected rate of inflation. This real interest rate (rate of return) is the rate of interest adjusted for the rate of inflation.

9. Discounting allows for comparing future costs and returns with current costs and returns.

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10. An agent would prefer an income stream with a higher present value.

11. The present value of a perpetuity yielding a return per time period is determined by dividing this return by the rate of return.

12. Net present value of an investment option is the discounted benefits and costs of the option. If the future net benefits are known with certainty, then the option with the highest NPV should be undertaken first and all independent options with NPV > 0 should be undertaken.

13. (Appendix) The discounted present value of future income is the value of this future income if it is instead received in the current period.

14. (Appendix) A lender is always a lender for any increase in interest rates. A borrower is always a borrower for any decrease in interest rates.

15. (Appendix) The no-arbitrage condition is the application of the Law of One Price to investment decisions. Identical assets must yield the same rate of return.

16. (Appendix) With uncertainty, sunk cost, and postponement, a hurdle rate triggers an investment decision. This hurdle rate may require that the present value of benefits exceed the costs by two or three times before an asset will be purchased.

17. (Appendix) The option of making an investment decision has some value (option value), which is lost once the decision to invest is undertaken. Considering this option value results in a positive hurdle rate, so ENPV must be greater than the option value before investment should occur.

18. (Appendix) For utility-based investments, the rate of return is composed of the utility rate of return plus the appreciation rate of return.

**Key Concepts**

- appreciation
- call option
- consol
- discount rate
- durable input (commodity)
- human capital
- human-capital production function
- hurdle rate
- hyperbolic discounting
- impatient
- interest rate
- intertemporal choice
- investment
- liquidity
- locked-in
- marginal rate of time preference
- net present value criterion
- nominal interest rate
- nondurable input (commodity)
- option value
- patient
- present value (PV)
- rate of return
- real option value

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Key Equations

\[ x_2 = I_2 + (1 + i)(I_1 - x_1) \]
Intertemporal budget constraint.

\[ \text{MRS}(x_2 \text{ for } x_1) = \frac{\partial U / \partial x_2}{\partial U / \partial x_1} = 1 + i \]
The equilibrium levels of current and future consumption occur where the marginal rate of time preference is set equal to the time rate of substituting future consumption for current consumption.

\[ \text{MRS}(x_2 \text{ for } x_1) = \text{MRPT}(x_2 \text{ for } x_1) = 1 + i \]
For the case with financial markets, intertemporal utility is maximized where the marginal rate of time preference and the marginal rate of product transformation are both set equal to the time rate of substituting future consumption for current consumption.

\[ 1 + r = \frac{1 + i}{1 + \alpha} \]
The additional consumption a consumer will receive in the future if the consumer reduces its current consumption by a marginal unit is equal to the additional future income the consumer would receive for this marginal reduction in current consumption divided by the future price level.

\[ \text{PV} = \frac{\text{TR}}{r} \]
The present value of a perpetuity yielding TR per period is determined by dividing this return by the rate of return.

\[ \frac{\partial x_1}{\partial i} \bigg|_{dU=0} = \frac{\partial x_1}{\partial i} + (I_1 - x_1) \frac{\partial x_1}{\partial I} \]
Adjusted Slutsky equation for a change in the interest rate.

\[ r = \frac{U + \text{TR}}{p} \]
For utility-based investments, the rate of return is equal to the monetary value of utility from owning the capital commodity plus the amount of appreciation divided by the purchase price.

real interest rate
riskless arbitrage
salvage value
Separation Theorem
sunk cost
TESTING YOURSELF

Multiple Choice

1. A consumer’s current and future consumption levels depend on
   a. Current and future income
   b. The interest rate
   c. Current and future prices
   d. All of the above.

2. The rate a consumer is willing to substitute future consumption for current consumption is called the marginal rate of
   a. Transformation
   b. Consumption
   c. Time preference
   d. Substitution.

3. Suppose Erica can earn an income of $100,000 this year and with a rise will earn $110,000 next year. If the interest rate is 7 percent, her intertemporal budget constraint can be represented by
   a. \( x_2 = I_2 + (1 + i)(I_1 - x_1) \)
   b. \( x_1 = I_1 + (1 + i)(I_2 - x_2) \)
   c. \( x_1 = I_1 - (1 + i)(I_2 - x_2) \)
   d. \( x_2 = I_2 + (1 - i)(I_1 - x_1) \).

4. The slope of an intertemporal budget constraint (assuming there are only two periods) is equal to
   a. \(- (1 - i)\)
   b. \(- (1 + i)\)
   c. \(- (I_1 + i)\)
   d. \(- (I_1 + x_1)\).

5. If a consumer has strictly convex intertemporal preferences,
   a. \(\text{MRS}(x_2 \text{ for } x_1)\) rises as \(x_1\) increases
   b. \(\text{MRS}(x_2 \text{ for } x_1)\) falls as \(x_1\) increases
   c. The consumer is impatient
   d. The consumer will be patient at low values of \(x_1\) and impatient at high values of \(x_1\).

6. Assuming a positive interest rate, the optimal level of current and future consumption occurs where the
   a. Slope of the budget constraint is equal to the negative of the interest rate
   b. Marginal rate of time preference is equal to the interest rate

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c. Slope of the indifference curve is equal to MRS($x_2$ for $x_1$)
d. Consumer is impatient.

7. A consumer will be a lender if ____________ at the Polonius point.
   a. MRS < 1 + $i$
   b. MRS > 1 + $i$
   c. MRS = 1 + $i$
   d. None of the above.

8. Which will not increase a worker’s human capital?
   a. College education
   b. On-the-job training
   c. Vocational training
   d. None of the above.

9. The human-capital production function is
   a. Convex
   b. Linear in education
   c. Increasing in education
   d. Concave.

10. In the absence of a financial market, the optimal level of human capital occurs where
    a. MRS($x_2$ for $x_1$) = MRPT($x_2$ for $x_1$)
    b. MRPT($x_2$ for $x_1$) = 1 + $i$
    c. MRS($x_2$ for $x_1$) = 1 + $i$
    d. Both b and c.

11. The Separation Theorem implies that a
    a. Consumer will separate investment into current and future consumption
    b. Consumer will separate her income and consumption
    c. Financial market allows consumers to separate production and consumption decisions
    d. Financial market allows consumers to separate current and future consumption.

12. If a financial market is present, which of the following describe the optimal level of human-capital investment?
    a. MRS($x_2$ for $x_1$) = MRPT($x_2$ for $x_1$)
    b. MRPT($x_2$ for $x_1$) = 1 + $i$
    c. MRS($x_2$ for $x_1$) = 1 + $i$
    d. Both b and c.
13. As uncertainty about the future rises, the ___________ future consumption ________ and the interest rate ________.
   a. Demand for, rises, rises
   b. Supply of, rises, rises
   c. Demand for, falls, rises
   d. Demand for, falls, falls.

14. Suppose the nominal rate of interest is \( i \) and the rate of inflation is \( \alpha \). The real rate of return, \( r \), will be equal to
   a. \( (i - \alpha)/(1 - \alpha) \)
   b. \( (i + \alpha)/(1 - \alpha) \)
   c. \( (i + \alpha)/(1 + \alpha) \)
   d. \( (i - \alpha)/(1 + \alpha) \).

15. If the interest rate is 10 percent, the present value of $100 paid next year is
   a. $110
   b. $100
   c. $91
   d. $90.

16. Russell is a professional basketball player and just renewed his contract guaranteeing him $10 million annually for the next five years. If the interest rate is 10 percent, the present value of this contract is
   a. $45.4 million
   b. $37.9 million
   c. $50 million
   d. $40.5 million.

17. The present value of a $500 perpetuity at an interest rate of 5 percent is
   a. $10,000
   b. $5000
   c. $25,000
   d. $100,000.

18. The net present value criterion suggests that an investment should be undertaken as long as
   a. The present value of the expected future cash flow is larger than the cost of the investment
   b. The future value of the expected cash flows is larger than the cost of the investment
   c. Financing can be secured
   d. There are no capital constraints.

19. Young’s Market is considering a $130,000 purchase of new machinery that will increase the market’s profits by $50,000 annually over the next three years. After three years, the machines will be obsolete and have no salvage value. What is the net present value of this
investment if the interest rate is 10%?
   a. −$5,657
   b. $12,483
   c. $5,657
   d. −$12,483.

20. The following table lists the income streams for three different investment projects for a small California software firm:

<table>
<thead>
<tr>
<th>Option</th>
<th>Present</th>
<th>One year</th>
<th>Two years</th>
<th>Three years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renovate the labs</td>
<td>−$100</td>
<td>$500</td>
<td>$500</td>
<td>$500</td>
</tr>
<tr>
<td>Move to the Southeast</td>
<td>−50</td>
<td>1000</td>
<td>500</td>
<td>200</td>
</tr>
<tr>
<td>Move to India</td>
<td>−200</td>
<td>300</td>
<td>600</td>
<td>1200</td>
</tr>
</tbody>
</table>

If the interest rate is 8 percent, the software firm will
   a. Invest in all three, given they all have a positive net present value
   b. Move to the Southeast
   c. Renovate the labs
   d. Move to India.
**Short Answer**

1. Define the marginal rate of time preference. What does it mean for this value to be greater than one? Less than one?

2. Explain why intertemporal indifference curves are generally strictly convex?

3. Suppose Kelly’s income this year is $200 and is expected to be $300 next year. If the interest rate is 8 percent, illustrate Kelly’s intertemporal budget constraint. Assume the price of the commodity is equal to 1 in both periods. Illustrate what happens when the interest rate falls to 6 percent?

4. What condition must be met for a household to maximize its utility when considering the choice between current and future consumption? Explain.

5. Comment on the statement “At the optimal choice between current and future consumption, a consumer will be impatient.”

6. Suppose an individual can earn an income $I_1$ without any investment in human capital. Human capital can be produced with a production function that exhibits diminishing marginal returns in terms of increases in future income. Given no financial markets, illustrate graphically an individual’s optimal level of human-capital investment. What condition must be met?

7. Why would we expect the human-capital production function to be concave?

8. Explain the Separation Theorem.

9. Illustrate graphically how the interest rate is determined.

10. A firm is considering an investment project. Explain how the net present value method should be used to determine whether the project is accepted or not. Should the firm use the nominal or real interest rate in making this determination? Explain.
Problems

1. Suppose Chris’s intertemporal utility function is \( U = x_1^{1/2}x_2^{1/2} \). Demonstrate that this utility function will yield convex indifference curves. If \( x_1 = 4 \) and \( x_2 = 9 \), what are the level of utility and \( MRS(x_2 \text{ for } x_1) \)? Is Chris patient or impatient? Suppose, instead, \( x_1 = 9 \) and \( x_2 = 4 \). Does this change the previous answers?

2. Kathleen’s intertemporal utility function is \( U = x_1^{2/3}x_2^{1/3} \). Her income is $500 in the first period and $800 in the second period. If the interest rate is 6 percent, find Kathleen’s optimal levels of current and future consumption. Is Kathleen a lender or a borrower? Explain.

3. Kala’s intertemporal utility function is \( U = x_1^{1/2}x_2^2 \). Her income is $15 in the first period and $20 in the second period. At a 5 percent interest rate, find Kala’s optimal levels of current and future consumption. Is Kala a lender or a borrower? Explain.

4. Refer to Problem 3. Suppose Kala can produce human capital with the production function \( 4x_1^2 + 2x_2^2 = 2000 \), where 2000 represents the value of the Polonius point on the production function. Assuming no financial market is available; find Kala’s optimal levels of current and future consumption. What is her optimal level of human-capital investment?

5. If the nominal interest rate is 18 percent and the rate of inflation is 9 percent, what is the real interest rate?

6. You have won a talent contest and get to choose between two prizes. The first is $200 today and $200 one year from now. The second is $100 today and $310 one year from now. At what interest rate would you be indifferent between the two prizes?

7. Andrea has invested $1000 in an account that earns a real rate of return of 10 percent. How many years will it take until the value of the account has doubled?

8. The Kelly Corporation has an opportunity to invest $1.5 million in a project that will generate profits of $0.5 million at the end of the first year, $0.55 million at the end of the second year, $0.6 million at the end of the third year, and $0.625 million at the end of the fourth year. The salvage value of the investment is zero at the end of the fourth year. Calculate the net present value of this investment assuming that the interest rate is 9 percent. Should it be undertaken? Explain.